Int. J. Heat Mass Transfer. Vol. 7, pp. 229–238. Pergamon Press 1964. Printed in Great Britain

THE EFFECT OF RADIATION ON FILM-BOILNG HEAT TRANSFER

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(*Received* 3 *April* 1963 *and in revised form* 28 *June* 1963)

Abstract—An analysis has been performed to determine the extent to which film-boiling heat transfer is affected by thermal radiation. Consideration is given both to the direct radiation between the heated surface and the liquid and to the emission and absorption of radiation in the vapor layer which lies between the surface and the liquid. It is found that the direct surface-to-liquid radiation can appreciably increase the film-boiling heat transfer. A quantitative criterion is deduced which states the conditions under which the effects of surface-to-liquid radiation are significant. The analysis which includes the radiatively-participating vapor involves some uncertainty in that only incomplete information is available on the radiation properties of vapors. In particular, even for steam, the vapor emissivity is not known with any certainty at pressures above 1 atm. The results of the analysis indicate that the effects of a radiatively-participating vapor on heat transfer are negligible within the parameter range investigated.

NOMENCLATURE

B, radiosity, energy/time-area;
c_p, specific heat, constant pressure;
E, attenuation factors, equation (16a);
g, acceleration of gravity;
h, heat-transfer coefficient;
k, thermal conductivity;
$\dot{m}$, vapor flow rate;
N_1, conductive–radiative parameter, equation (6b);
N_2, conductive–radiative parameter, equation (22);
Pr, Prandtl number, $c_p\mu/k$;
p, pressure;
Q, overall heat-transfer rate;
q, local heat flux rate;
T, absolute temperature;
T_w, temperature of heated wall;
T_s, liquid saturation temperature;
u, longitudinal velocity component;
X, dimensionless co-ordinate, $\left[\frac{g\rho_L\lambda^*}{16\nu\kappa(T_w - T_s)}\right]^{1/3} x$;
x, longitudinal co-ordinate;
Y, dimensionless co-ordinate, y/δ;
y, transverse co-ordinate;
Δ, dimensionless thickness, $\left[\frac{g\rho_L\lambda^*}{16\nu\kappa(T_w - T_s)}\right]^{1/3} \delta$;
δ, vapor layer thickness;
δ^*, equivalent beam length;
ϵ, emissivity;
θ, dimensionless temperature, T/T_S;
κ, absorption coefficient;
λ, latent heat;
λ^*, modified latent heat, equation (1);
μ, absolute viscosity;
ν, kinematic viscosity;
ρ, density;
σ, Stefan–Boltzmann constant;
τ, optical thickness.

Subscripts
c, conductive–convective;
L, liquid;
o, in the absence of radiation;
r, radiative;
w, heated wall.

INTRODUCTION

UNDER film-boiling conditions, the rate of heat transfer from a heated surface to a boiling liquid is affected both by convective–conductive transport and by radiative transport. The radiative heat transfer becomes increasingly more important as the surface temperature increases relative to the bulk liquid temperature. As is indicated on typical boiling curves (e.g. [1], Fig. 9.1), the effect of the radiation is to increase the boiling heat transfer with increasing values of wall-to-bulk temperature difference.

There are two processes by which the radiative transport may affect the film-boiling heat transfer. The first is a direct transfer between the heated surface and the liquid. The second is the absorption and emission of radiation in the vapor film which lies between the surface and the liquid. In some cases, the vapor (or gas) may have no absorption or emission bands which lie in the wave length range of importance to radiative heat transport; correspondingly, the only radiation effect is the direct transfer from surface to liquid. Typical non-participating gases are oxygen, hydrogen, nitrogen, and so forth. Many vapors, including steam, have absorption and emission bands in the thermal range ([2], pp. 82–92). When such a participating vapor is present, it may be effective in modifying the distribution of temperature across the vapor film; by this, there is created a coupling between the radiative and the convective-conductive energy transport.

The problem of film boiling in the presence of a radiatively non-participating vapor has been previously considered by Bromley [3]. As discussed in his paper, Bromley attempted to incorporate the effects of surface-to-liquid radiation into the energy equation for the conductive–convective transport. The resulting differential equation appeared not to be readily solvable. As an alternative, he used qualitative arguments to derive a heat-transfer coefficient for the combined convection and radiation process.

The effect of a radiatively-participating vapor on film-boiling heat transfer has not been previously studied within the knowledge of the present author.

The present paper is concerned with the effects of radiation on film-boiling heat transfer. The paper is divided into two parts. In the first section, consideration is given to the case of film boiling in the absence of a radiatively-participating vapor. The presence of a participating vapor will be considered in the second section of the paper. The analysis is carried out for stable laminar film boiling on a vertical isothermal plane surface immersed in a large body of liquid. This physical configuration is shown diagramatically in Fig. 1. The liquid is taken to

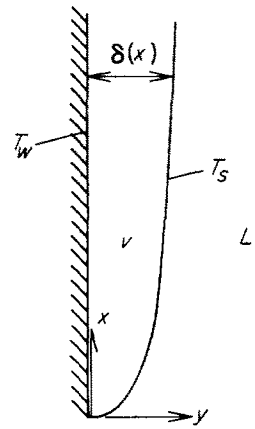

FIG. 1. Physical model and co-ordinates.

be at its saturation temperature T_s, while the wall temperature is uniform and is denoted by T_w. Forced-convection motions are absent. For purposes of analysis, the vapor film is assumed to be smooth; the film thickness, denoted by δ, may vary with the longitudinal co-ordinate x.

CONVECTIVE–RADIATIVE HEAT TRANSFER, NON-PARTICIPATING VAPOR

As a point of departure for the study of radiative effects on film boiling, it is worthwhile to briefly mention some pertinent findings from prior analyses of the convective film-boiling problem. The initial analysis among these appears to be due to Bromley [3], who employed a simple model which neglected inertia forces and superheating within the vapor film and additionally assumed a zero longitudinal vapor velocity at the liquid–vapor interface. Recently, more complete analytical treatments [4–6] have been carried out which eliminate the aforementioned simplifying assumptions. It has been shown that the effects of the vapor inertia forces and superheating could essentially be eliminated from the problem by defining a modified latent heat of vaporization λ^* in terms of the actual latent heat λ, the temperature difference $(T_w - T_s)$, and the vapor properties

$$\lambda^* = \lambda\,[1 + 0{\cdot}84\, c_p(T_w - T_s)/\lambda Pr] \qquad (1)$$

The assumption of zero interfacial velocity leads to a slight under-prediction of the heat-transfer results, the importance of which decreases with increasing values of $c_p(T_w - T_s)/\lambda Pr$, i.e. at higher rates of evaporation. For operating conditions which correspond to stable film boiling, the under-prediction appears to be on the order of 5–10 per cent. This level of uncertainty in the analysis is not of great practical importance since the realities of the film-boiling process, such as instabilities and surface ripples, can contribute to much greater disparities between theory and experiment.

In light of the foregoing discussion, it appears that Bromley's model for the convective film-boiling problem is quite adequate, provided that the latent heat is modified according to equation (1). This model will be applied and extended here for the radiative–convective film-boiling problem.

The starting point of the analysis is the conservation laws. In the absence of the inertia terms, momentum conservation reduces to a force balance

$$-\frac{\partial p}{\partial x} - \rho g + \mu \frac{\partial^2 u}{\partial y^2} = 0 \qquad (2a)$$

in which unsubscripted properties correspond to those of the vapor. According to boundary layer theory, the pressure gradient $\partial p/\partial x$ is uniform across the layer (i.e. independent of y), therefore

$$\frac{\partial p}{\partial x} = -\rho_L g \qquad (2b)$$

where ρ_L is the density of the liquid. After eliminating the pressure between the foregoing and integrating the velocity equation subject to the boundary conditions that $u = 0$ at $y = 0$ and $y = \delta$, one finds†

$$u = \frac{\rho_L g}{2\mu}\,\delta^2\,[(y/\delta) - (y/\delta)^2]. \qquad (3)$$

† In this, it has been assumed that $\rho \ll \rho_L$.

Additionally, the mass rate of vapor flowing through a typical cross section of boundary layer of thickness δ is

$$\dot{m} = \int_0^\delta \rho u\, dy = (\rho \rho_L g/12\mu)\delta^3. \qquad (4)$$

From this, it follows that the increment of mass added to the boundary layer by vaporization in a length dx is

$$\frac{d\dot{m}}{dx} = (\rho \rho_L g/4\mu)\delta^2 \frac{d\delta}{dx}. \qquad (4a)$$

Consideration may now be given to energy conservation. If superheating of the vapor is temporarily put aside (it will be included later as a correction), then the temperature profile is linear, i.e. $T = T_w + (T_s - T_w)(y/\delta)$. The local heat transfer at a surface location is made up of two components, a conductive–convective component $q_c = -k(\partial T/\partial y)$ and a radiative component q_r. In formulating the radiative component, cognizance is taken of the fact that the vapor layer is very thin. Because of this, the radiant interchange between a surface element and the liquid occurs between locations which are essentially opposite to one another. Consequently, the non-parallelism of the surface and the interface does not affect the local radiant heat transfer and one can use the interchange factors appropriate to two parallel planes, thus

$$q_r = h_r(T_w - T_s),$$
$$h_r = \frac{\epsilon_w \epsilon_L}{\epsilon_L + \epsilon_w - \epsilon_w \epsilon_L}\,\frac{\sigma(T_w^4 - T_s^4)}{(T_w - T_s)}. \qquad (5)$$

The emissivity ϵ_L of the liquid surface is essentially unity.

The local surface heat transfer q is responsible for creating additional amounts of vapor at the rate dm and superheating it. The superheating will be accounted for by use of the modified latent heat λ^*. It then follows that

$$q = q_c + q_r = \lambda^*(d\dot{m}/dx). \qquad (6)$$

Upon evaluating q_c, q_r, and d$\dot{m}$ from the foregoing, one has

$$q = \frac{k(T_w - T_s)}{\delta} + h_r(T_w - T_s) = (\rho\rho_L g \lambda^*/4\mu)\delta^2\,(d\delta/dx). \qquad (6a)$$

A dimensionless counterpart of this equation is

$$\mathrm{d}X = \frac{4\Delta^3}{1 + N_1\Delta}\,\mathrm{d}\Delta,$$

$$N_1 = \frac{h_r}{k}\left[\frac{16\nu k(T_w - T_s)}{g\rho_L\lambda^*}\right]^{1/3}. \quad (6b)$$

The parameter N_1 is a measure of the relative importance of the radiative and the conductive–convective transports. $N_1 = 0$ corresponds to the case of negligible radiation. As N_1 increases, the radiative transport is increased, but to an extent which will be determined by the forthcoming solutions. It is readily seen from equation (6b) that at a given location x, the thickness of the vapor layer is greater due to the presence of radiation.

A closed-form solution of equation (6b) may be found as follows

$$\frac{4}{N_1^4}\left[\frac{N_1^3}{3}\Delta^3 - \frac{N_1^2}{2}\Delta^2 + N_1\Delta - \ln(1 + N_1\Delta)\right] = X. \quad (7)$$

For $N_1 \to 0$, it is easily verified (by using the series expansion for the logarithm) that this reduces to the solution which applies in the absence of radiation

$$\Delta_o = X^{1/4} \quad (8)$$

in which the subscript o denotes purely conductive–convective transport.

With the solution for Δ in hand, one may return to equation (6a) and calculate the local heat transfer q. An informative presentation of the heat-transfer results may be made in terms of the ratio of q to q_o, the latter representing the case of purely conductive–convective transport. Such a presentation provides a direct measure of the effect of the radiation. In addition, the ratioing of the results may cancel out errors introduced by the simplifying assumptions of the analysis. From equation (6a) in conjunction with (8), one finds

$$\frac{q}{q_o} = \frac{X^{1/4}}{\Delta}(1 + N_1\Delta). \quad (9)$$

The overall rate of heat transfer Q from a section of plate from $x = 0$ to $x = x$ may also be determined.

$$Q = \int_0^x q\,\mathrm{d}x. \quad (10)$$

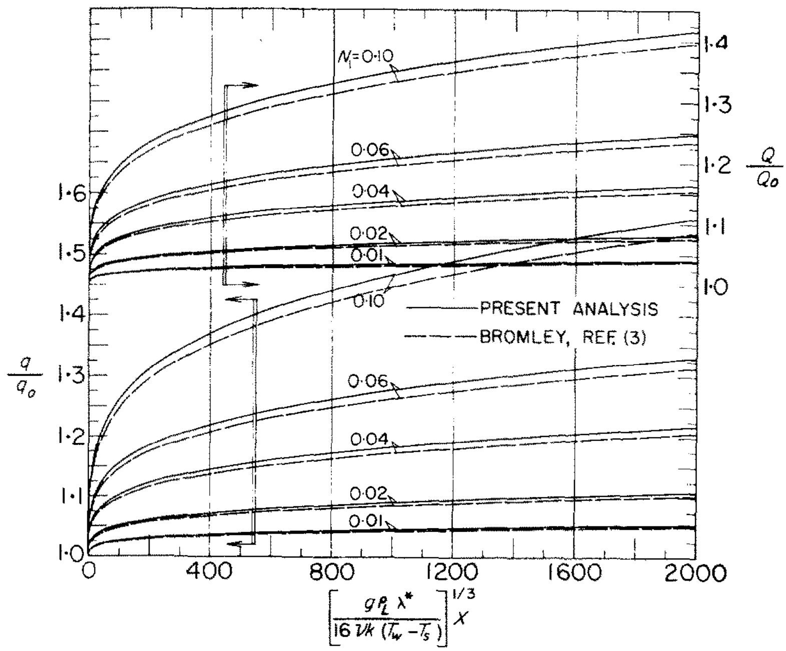

FIG. 2. Effect of surface-to-liquid radiation on film-boiling heat transfer, range of smaller N_1.

Upon integrating the local heat flux, one finds

$$Q/Q_o = \Delta^3/X^{3/4} \qquad (11)$$

in which Q_o is the overall heat transfer for purely conductive–convective conditions. As presented in this ratio form, the Q/Q_o should also serve as an estimate for the effects of radiation in other configurations besides the vertical plate.

Numerical values of q/q_o and Q/Q_o have been evaluated from equations (9) and (11) and are presented in Figs. 2 and 3. The first of these corresponds to the lower values of N_1, while the second is for larger values of N_1. The lower part of each figure is devoted to the q/q_o results and utilizes the left-hand ordinate; the upper part of each figure is for the Q/Q_o results and uses the right-hand ordinate. The abscissa of both graphs is the dimensionless distance from the lower edge of the plate.

From the figures, there are several interesting trends which are evident. First of all, as expected, the effect of radiation is accentuated with increasing values of the parameter N_1. Additionally, the radiation becomes relatively more important at greater distances from the lower edge. In particular, there is a sharp increase of both q/q_o and Q/Q_o near $x = 0$ and a much more gradual increase thereafter. This finding is understood by recalling that the conductive–convective transport is extremely large near the lower edge of the plate where the vapor layer is very thin. In this region, the radiative contribution is over-ridden regardless of the value of N_1. As x increases, the vapor layer grows thicker and the conductive–convective transport decreases, thereby providing an opportunity for the radiation to become relatively more important. Further study of the figures reveals that the effect of radiation on Q/Q_o is less than on q/q_o. This is because the averaging process which calculates Q/Q_o includes upstream positions at which the effect of radiation is smaller than it is at x.

It is interesting to consider the operating conditions for which the contribution of radiation should be accounted. In light of experimental practice for film boiling, it is reasonable to regard deviations of Q from Q_o of 10–20 per cent as having practical significance. On this

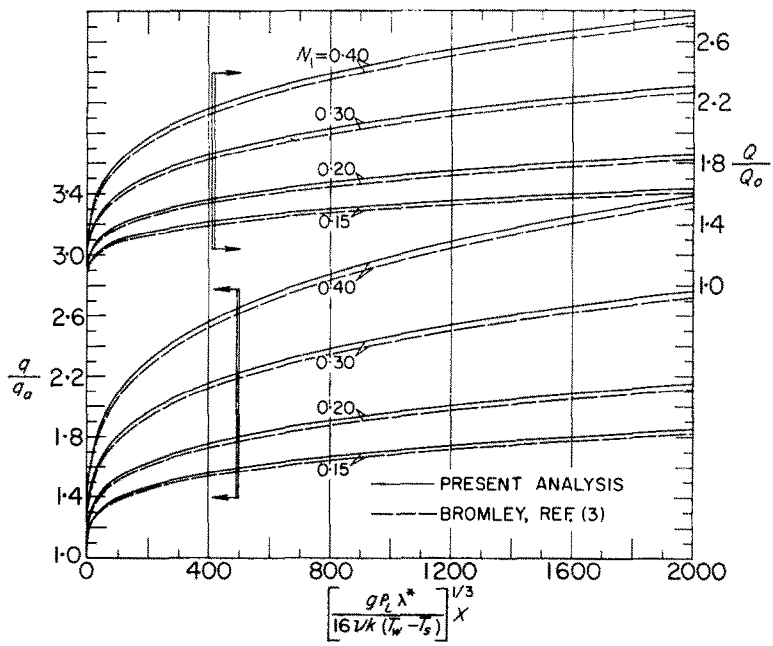

FIG. 3. Effect of surface-to-liquid radiation on film-boiling heat transfer, range of larger N_1.

basis, it is suggested that radiation be accounted for whenever $N_1 \geqslant 0{\cdot}04$, that is, when

$$h_r \geqslant 0{\cdot}04\, k \left[\frac{g\rho_L \lambda^*}{16\nu\kappa(T_w - T_s)}\right]^{1/3}. \qquad (12)$$

Turning to Fig. 3, it is seen that very substantial deviations of q/q_o and Q/Q_o from unity may occur at the larger values of the N_1 parameter. As a point of interest, we have calculated that N_1 values up to 0·20 occurred in Bromley's experiment (3) involving steam.

It is also of interest to compare the present results with those derived by Bromley using intuitive arguments. Bromley gives a formula for the heat-transfer coefficient h for simultaneous convective–radiative transport in terms of the coefficients h_o for purely conductive–convective transfer and h_r for purely radiative transfer

$$h = h_o\,(h_o/h)^{1/3} + h_r. \qquad (13)$$

It is not indicated whether h is to be regarded as a local coefficient or an overall coefficient. If Bromley's h and h_o are taken to be local coefficients, then equation (13) can, after some manipulation, be recast into the following form applicable to the vertical plate

$$\left\{\frac{1}{N_1}\left[\left(\frac{q}{q_o}\right) - \left(\frac{q_o}{q}\right)^{1/3}\right]\right\}^4 = X. \qquad (13a)$$

Alternatively, if the h and h_o are meant to represent overall coefficients, then a form of equation (13) applicable to the vertical plate is

$$\left\{\frac{4}{3N_1}\left[\left(\frac{Q}{Q_o}\right) - \left(\frac{Q_o}{Q}\right)^{1/3}\right]\right\}^4 = X. \qquad (13b)$$

The foregoing equations (13a) and (13b) have been numerically evaluated, and the results are plotted as dashed lines in Figs. 2 and 3. Comparison of these results with those of the present, more complete, analysis shows good agreement. Generally, the dashed curves lie slightly lower than the solid curves, but the deviations are no more than a few per cent. Considering that Bromley himself characterized the formulation leading to equation (13) as being no more than qualitatively correct, the agreement indicated by Figs. 2 and 3 must be regarded as truly remarkable.

CONVECTIVE–RADIATIVE HEAT TRANSFER, PARTICIPATING VAPOR

In considering the effect of a radiatively-participating vapor, it is natural to begin with the emission and absorption properties of the vapor. Of all the radiatively-participating vapors, steam is perhaps of the greatest practical importance. The radiation properties of steam. although in themselves incomplete (especially in regard to pressure dependence), are more fully known than those of any other vapor of possible interest in the boiling problem. The most complete presentation of the radiation properties of steam is given by Hottel [2, Chapt. 3]. For vapor film thickness which is typical of film boiling ($\delta \sim 0{\cdot}01$–$0{\cdot}02$ in), the emissivity of steam at atmospheric pressure appears to be less than 0·01. The effect of such an emissivity on the heat transfer is completely negligible (to be demonstrated later).

The emissivity of steam increases with increasing pressure level. Unfortunately, at pressures in excess of 1 atm, numerical values for the emissivity are not available. The difficulty lies in the absence of information on the so-called line broadening effect, which relates to the broadening of the spectral lines at increasing pressure. Bevans [7] sounds a strong note of caution against extrapolating Hottel's line-broadening correction [2, Figs. 4–16] to higher pressure.

In order to get a numerical estimate of the effects of a radiatively-participating vapor on heat transfer, consideration will be given here to a vapor whose emissivity can range as high as 0·2. This might perhaps correspond to steam at a pressure of 10 atm.

The formulation of the heat-transfer problem involving a participating vapor utilizes and extends the ideas previously introduced for the case of the non-participating vapor. In particular, consideration is given to a thin vapor layer of thickness δ in which the surface and interface are effectively parallel and in which superheating of the vapor is neglected. The first step in the analysis is to apply energy conservation to a differential volume element within the vapor. In the steady state, the sum of the net heat conduction and the net radiative transport out of the element must be zero, thus

$$(\mathrm{d}q/\mathrm{d}y)_{\mathrm{cond}} + (\mathrm{d}q/\mathrm{d}y)_{\mathrm{rad}} = 0. \quad (14)$$

Because the vapor layer is thin, longitudinal transport (in the x-direction) is negligible relative to the transverse transport (in the y-direction).

The first term of equation (14) reduces to the familiar heat conduction form

$$(\mathrm{d}q/\mathrm{d}y)_{\mathrm{cond}} = -k(\mathrm{d}^2T/\mathrm{d}y^2). \quad (15)$$

The derivation of the radiation term is quite lengthy and only a general discussion will be given here (interested readers may consult [8] for further details). The net radiant outflow from the element includes four contributions: (a) the radiant emission from the element, (b) the absorption of radiant energy initially emitted by all other elements throughout the vapor, (c) the absorption of radiant energy which comes from the heated wall and (d) the absorption of radiant energy which comes from the liquid surface. In connection with items (b), (c) and (d), cognizance is to be taken of the attenuation which may be experienced by radiation as it traverses the distance between its source and the control volume. Additionally, in connection with (c) and (d), it may also be noted that if a surface is non-black, the radiant energy leaving the surface includes not only the emission, but also the reflected portion of the incident radiation. In particular, for non-black surfaces which are diffuse emitters and reflectors, it is convenient to work with the sum of the emitted and reflected radiation; this sum is often called the radiosity and may be denoted by the symbol B.

When the aforementioned contributions to the net radiation are formulated in mathematical terms, there follows

$$\left(\frac{\mathrm{d}q}{\mathrm{d}y}\right)_{\mathrm{rad}} = 4\kappa\sigma T^4(y) - 2\kappa\left[\int_0^\delta \sigma T^4(y')E_1(|y-y'|)\,\mathrm{d}y' + B_w E_2(y) + B_L E_2(\delta - y)\right] \quad (16)$$

in which the attenuation factors, E_1 and E_2, are defined as

$$E_n(z) = \int_0^1 \omega^{n-2}\, e^{-z/\omega}\,\mathrm{d}\omega. \quad (16a)$$

The symbol κ denotes the absorption coefficient of the vapor, and B_w and B_L are respectively the radiosities of the heated wall and the liquid surface.

When the radiative and conductive portions of the energy balance are added together in accordance with equation (14), there results an equation for the distribution of the temperature T across the vapor layer. Inasmuch as the unknown T appears both under the integral sign and under the derivative operator, this is an integro-differential equation.

In the present problem, a significant simplification can be affected by taking cognizance of the fact that the vapor is a weak absorber and emitter of radiant energy. To explain the basis of this simplification, it is convenient to make use of the optical thickness τ,

$$\tau = \kappa\delta. \quad (17a)$$

For a weakly absorbing–emitting gas, Eckert and Drake [9, pp. 389–393] have shown that the optical thickness based on an equivalent beam length δ^* is quite simply related to the gas emissivity

$$\kappa\delta^* = \epsilon. \quad (17b)$$

Additionally, the equivalent beam length δ^* for a plane gas layer is [9, Table 13-4]

$$\delta^* = 1{\cdot}8\delta. \quad (17c)$$

From the foregoing, it follows that

$$\tau \approx \tfrac{1}{2}\,\epsilon. \quad (18)$$

For a gas emissivity on the order of 0·2, the corresponding optical thickness is 0·1. For optical thicknesses of this magnitude, Viskanta [10] has recently demonstrated that accurate results for the conductive–radiative heat transfer can be obtained by applying a simplified version of the governing integro-differential equation.

The simplified temperature equation for small τ is derived by noting that

$$\int_0^\tau E_1(y')\,\mathrm{d}y' \sim 0(\tau),$$
$$E_2(\tau) \sim 1 + 0(\tau), \quad E_3(\tau) \sim 0{\cdot}5 - \tau + 0(\tau^2). \quad (19)$$

With these, the governing equation for the temperature becomes

$$k\frac{\mathrm{d}^2T}{\mathrm{d}y^2} = 4\kappa\sigma T^4(y) - 2\kappa(B_w + B_L). \quad (20)$$

Table 1. Effect of a participating vapor on heat transfer

N_2/τ^2	ϵ_w	τ	θ_w	$q_{part}/q_{nonpart}$
100	1	0·1	1·25	0·992
100	1	0·1	1·5	0·989
100	1	0·1	1·75	0·985
100	1	0·1	2	0·981
100	1	0·1	2·5	0·973
100	1	0·05	1·25	0·993
100	1	0·05	1·5	0·990
100	1	0·05	1·75	0·988
100	1	0·05	2	0·985
100	1	0·05	2·5	0·981
100	0·5	0·1	1·25	1·004
100	0·5	0·1	1·5	1·004
100	0·5	0·1	1·75	1·005
100	0·5	0·1	2	1·005
100	0·5	0·1	2·5	1·006
100	0·5	0·05	1·25	1·003
100	0·5	0·05	1·5	1·004
100	0·5	0·05	1·75	1·004
100	0·5	0·05	2	1·004
100	0·5	0·05	2·5	1·005
50	1	0·1	1·25	0·985
50	1	0·1	1·5	0·981
50	1	0·1	1·75	0·976
50	1	0·1	2	0·971
50	1	0·1	2·5	0·962
50	1	0·05	1·25	0·988
50	1	0·05	1·5	0·985
50	1	0·05	1·75	0·982
50	1	0·05	2	0·980
50	1	0·05	2·5	0·975
50	0·5	0·1	1·25	1·007
50	0·5	0·1	1·5	1·008
50	0·5	0·1	1·75	1·008
50	0·5	0·1	2	1·009
50	0·5	0·1	2·5	1·009
50	0·5	0·05	1·25	1·006
50	0·5	0·05	1·5	1·007
50	0·5	0·05	1·75	1·007
50	0·5	0·05	2	1·007
50	0·5	0·05	2·5	1·007

The liquid surface behaves very nearly like a black body, thus $B_L = \sigma T_s^4$. Additionally, it can be shown that the radiosity of the heated surface is expressible (to the first order) as

$$B_w = \epsilon_w \sigma T_w^4 + (1 - \epsilon_w)\sigma T_s^4.$$

After substituting these into equation (20) and introducing dimensionless quantities, there is obtained

$$\frac{N_2}{\tau^2}\frac{d^2\theta}{dY^2} = \theta^4 - \tfrac{1}{2}\left[\epsilon_w\theta^4 + (2 - \epsilon_w)\right]. \quad (21)$$

The optical thickness τ has already been defined in equation (17a), while the parameter N_2 is given by

$$N_2 = k\kappa/4\sigma T_s^4. \tag{22}$$

The magnitude of N_2/τ^2 is an index of the importance of radiant transport in the vapor relative to the conductive transport; $N_2/\tau^2 = \infty$ corresponds to pure conduction and $N_2/\tau^2 = 0$ corresponds to pure radiative transport. The boundary conditions on the dimensionless temperature $\theta = T/T_s$ are: at $Y = 0$, $\theta = \theta_w$ and at $Y = 1$, $\theta = 1$. Once the temperature distribution has been found from the solution of equation (21), the heat flux q at the wall may be calculated.

$$q/\sigma T_s^4 = -4\left(\frac{N_2}{\tau^2}\right)\tau\left(\frac{d\theta}{dY}\right)_{Y=0} + \epsilon_w(\theta_w^4 - 1) + 2\epsilon_w\tau\left(1 - \int_0^1 \theta^4\, dY\right). \tag{23}$$

The solution of equation (21) for the temperature distribution and the subsequent calculation of the heat transfer from equation (23) requires the specification of four parameters: the optical thickness τ, the ratio N_2/τ^2, the temperature ratio $\theta_w = T_w/T_s$, and the wall emissivity ϵ_w. Optical thickness values of 0·1 and 0·05 were selected, and these correspond approximately to vapor emissivities of 0·2 and 0·1 [equation (18)]. Values of 50 and 100 were chosen for N_2/τ^2 on the basis of the aforementioned selections for τ and on reasonable estimates of the other physical quantities. θ_w was varied through the range from 1·25 to 2·5. ϵ_w was given values of 1·0 and 0·5.

It is interesting to compare the heat transfer in the presence of a participating vapor with that in the presence of a non-participating vapor. This information is given in the last column of Table 1. Inspection of the table indicates that the effect of the radiatively-participating vapor on the heat transfer is fully negligible within the parameter ranges studied here. As previously indicated, this should include steam at pressures at least up to 10 atm. It is perhaps possible that vapor participation may have a greater effect at very high pressures. But, further analysis of the heat-transfer problem should reasonably await the determination of vapor emissivities at the higher pressures.

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Zusammenfassung—Der Einfluss der Wärmestrahlung auf den Wärmeübergang beim Filmsieden wurde in einer Analyse untersucht. Dabei ist sowohl die direkte Strahlung zwischen beheizter Oberfläche und Flüssigkeit, als auch die Emission und Absorption der Strahlung in der Dampfschicht zwischen Oberfläche und Flüssigkeit berücksichtigt. Es zeigte sich, dass die direkte Strahlung von der Oberfläche zur Flüssigkeit den Wärmeübergang beim Filmsieden spürbar erhöhen kann. Ein quantitatives Kriterium wurde abgeleitet; es legt die Bedingungen fest, unter denen die Oberflächen-Flüssigkeitsstrahlung einen Einfluss hat. Die Analyse für die Strahlungswirksame Dampfschicht ist insofern unsicher, als nur unvollständige Daten für die Strahlungseigenschaften von Dämpfen verfügvar sind. Insbesondere ist die Dampfemission—selbst für Wasserdampf—für Drücke über 1 atm nicht mit Sicherheit bekannt. Die Ergebnisse der Analyse deuten an, dass der Einfluss der strahlenden Dampfschicht auf den Wärmeübergang im Bereich der untersuchten Parameter vernachlässigbar ist.

Аннотация—Проведено исследование с целью определения степени влияния теплового излучения на теплообмен при пленочном кипении. Рассмотрен непосредственный теплообмен между нагретой поверхностью и жидкостью, а также испускание и поглощение излучения слоем пара, расположенным между поверхностью и жидкостью. Установлено, что непосредственная передача тепла излучением от поверхности к жидкости может оказывать заметное влияние на теплообмен при пленочном кипении. Получено значение критерия, которое показывает, при каких условиях имеет место существенное влияние излучения от поверхности к жидкости. Отмечается некоторая неопределенность результатов исследования из-за отсутствия полных данных об излучающих свойствах паров. В частности, даже для водяного пара точно не известна излучательная способность при давлениях выше 1 атм. Результаты исследования показывают, что в диапазоне исследумых параметров пар оказывает пренебрежимо малое влияние на теплообмен.